Molodensky-Badekas

Reducing the Consequences of Parametric Correlation in the 7-Parameter Shift

Presented by Noel Zinn of ExxonMobil

at the 26 February 2004 meeting of the

Americas Petroleum Survey Group

Houston, Texas

Acknowledgements

- Web site with DMA Cartesian graphics at https://www. navigator.navy.mil/navigator/coordinate_datum_transformations.ppt
- Web site with NGA Molodensky Model http://earth-info.nima.mil/GandG/datums/wgsdt.html
- Trimble GPS Survey Planning Software http://www.trimble.com/planningsoftware.html
- Technical peer review and suggestions by Roger Lott (EPSG Geodesy Working Group) - no endorsement of author's opinions implied

Topics

- **Statement of the issues**
- Geographicals to Cartesians to Geographicals
- Heuristic explanation of correlation problem
- The Molodensky Model at the geocenter (Helmert)
- The derivation of a 7-parameter shift
- Monte Carlo correlation w.r.t. datum area
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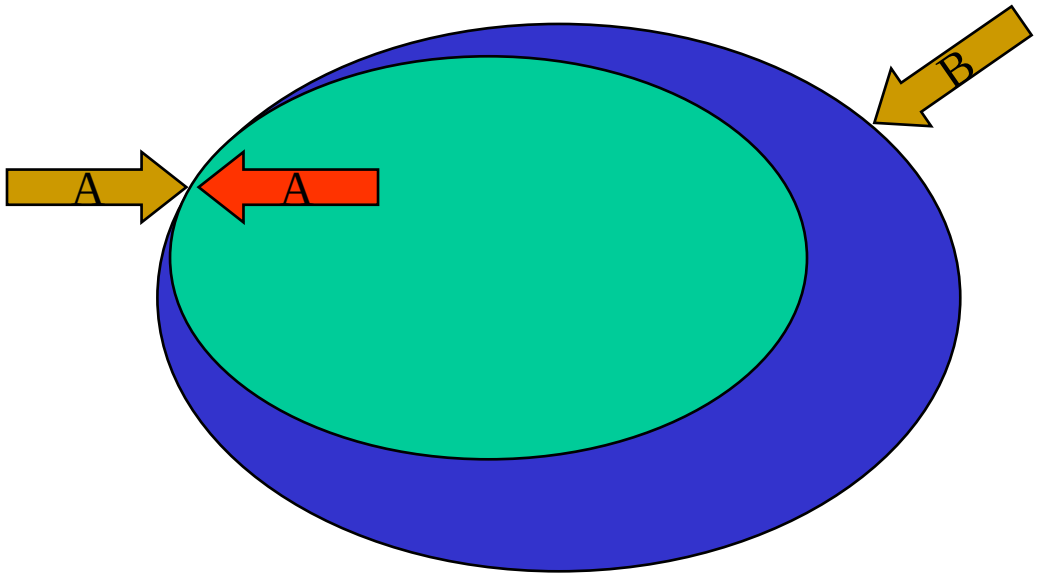
- Dilution of Precision (P7DOP)
- Conclusion
- Reversibility in Molodensky-Badekas (Appendix)

Transformation Error



The shift required depends on the location on the datum. It is not constant.

Issues

- 3-D 3-parameter shifts are perceived as not accurate enough, even in small datum areas.
- 2-D MRE and interpolation software (e.g., NADCON, NTv2) are not widely supported.
- 3-D 7-parameter Helmert shifts are supported.
- Do we derive 7-p Helmert shifts over small areas?
- Are these 7-p Helmert shifts really “better” than 3-p shifts in a small datum area? Or “worse”?
- Is Molodensky-Badekas a good alternative for deriving a 7-p shift in a small datum area?
- Reversibility in Molodensky-Badekas.

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Transformation Route

Geographical (ϕ/λ/ht) on Ellipsoid1

See Appendix

Geocentric Cartesian (X/Y/Z)

Helmert or M-B Transformation

Geocentric Cartesian (X'/Y'/Z')

See Appendix

Geographical (ϕ'/λ'/ht') on Ellipsoid2

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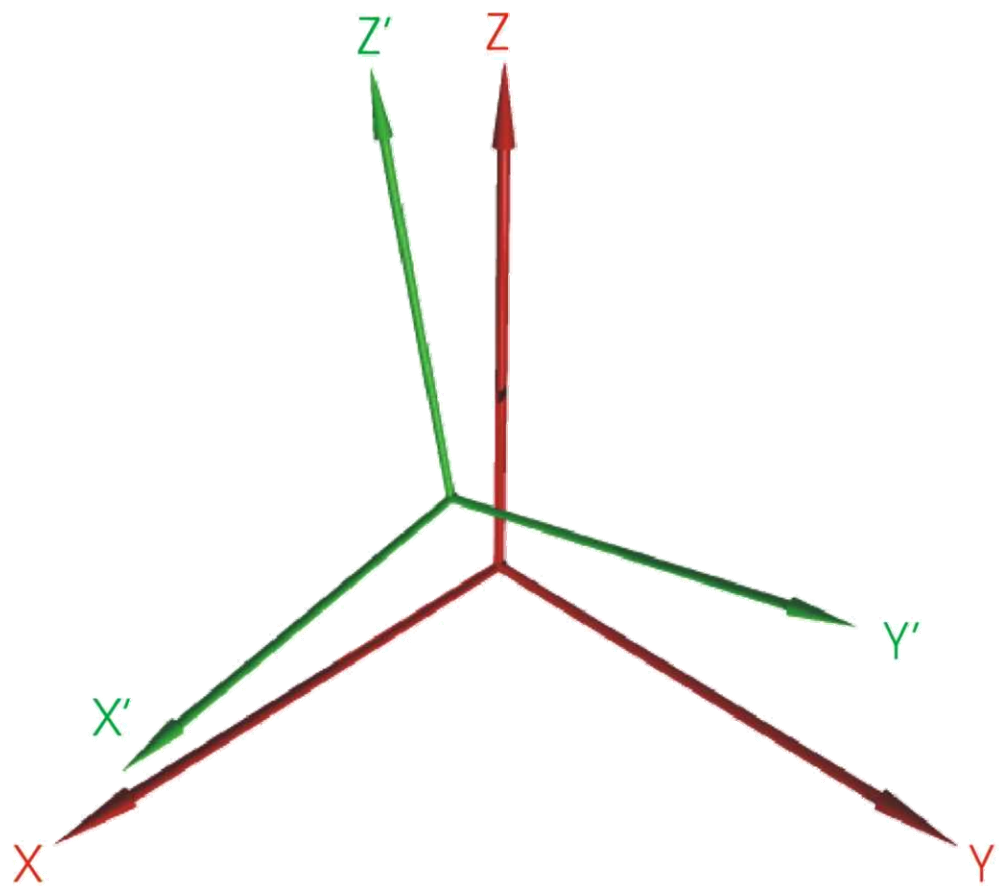
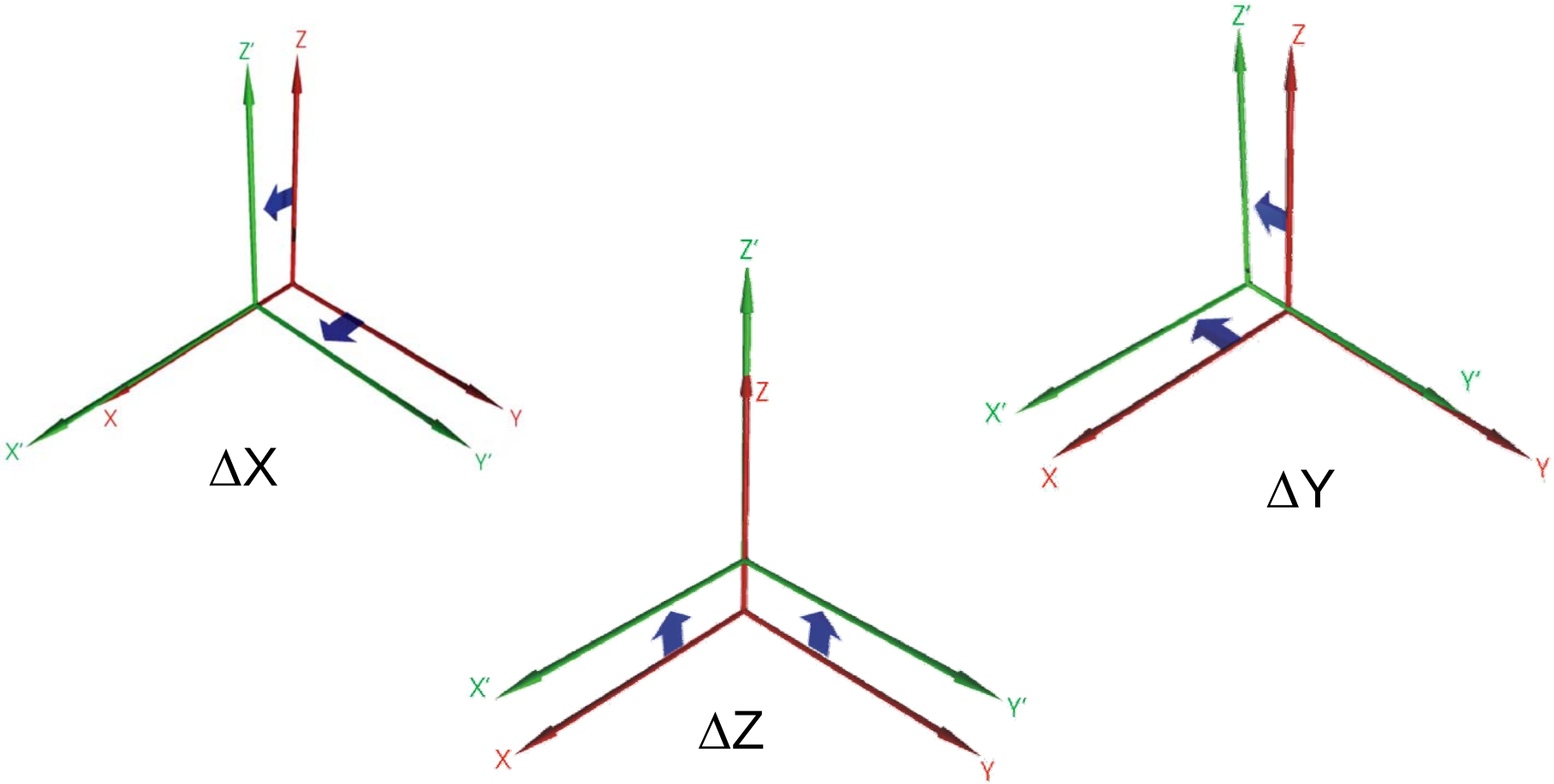
Datum Transformation



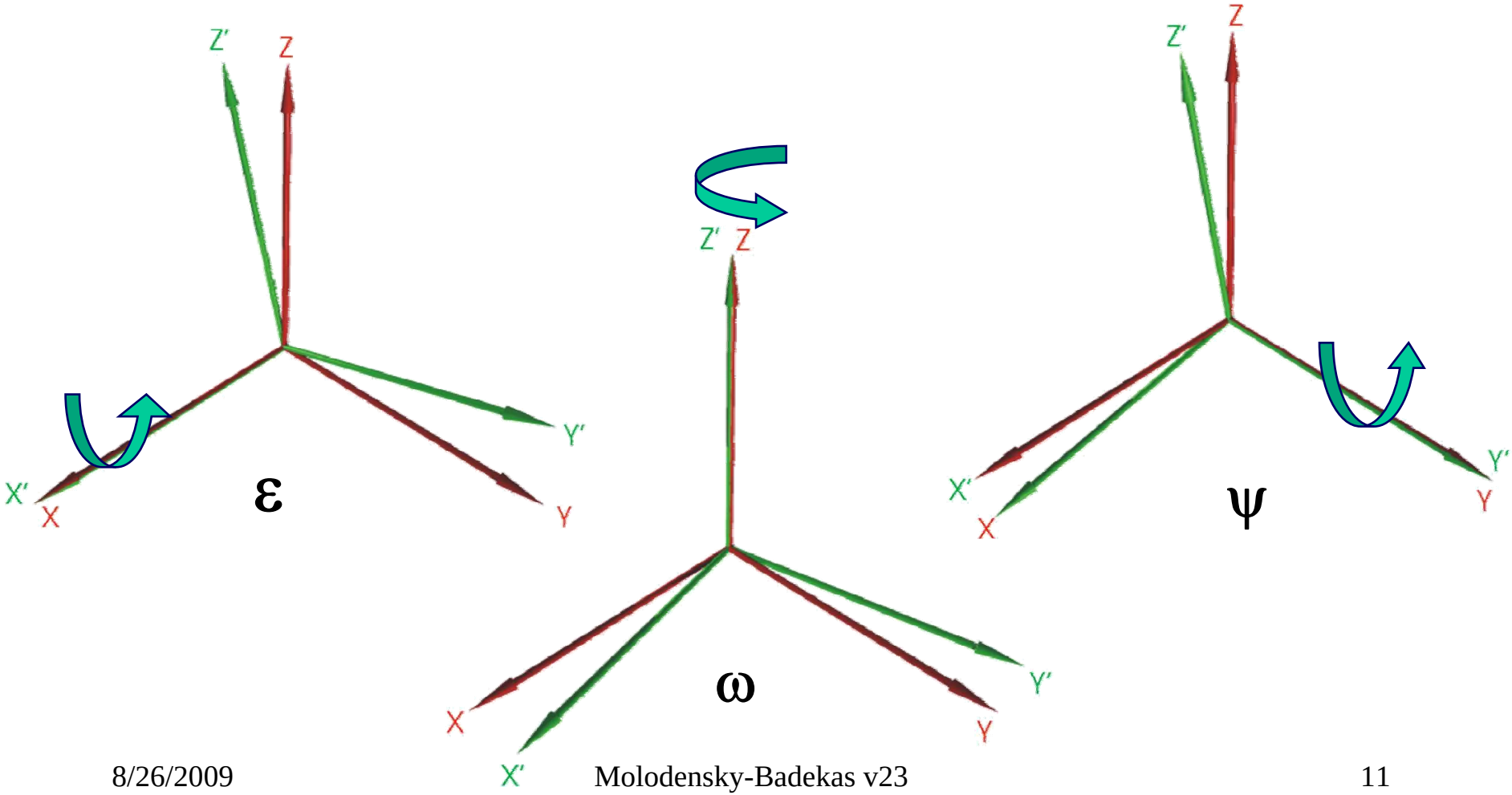
Translations (3 Parameters)

Movement of points <u>along</u> an Axis



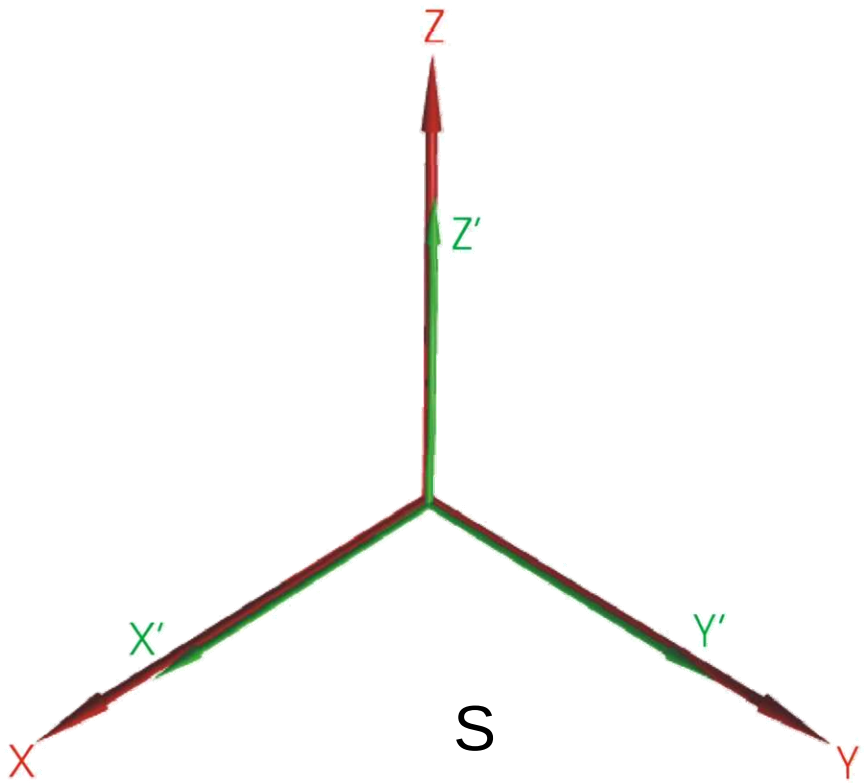
Rotations (3 Parameters)

Movement of points <u>around</u> an Axis



Scale (1 Parameter)

Changing the distance <u>between</u> points



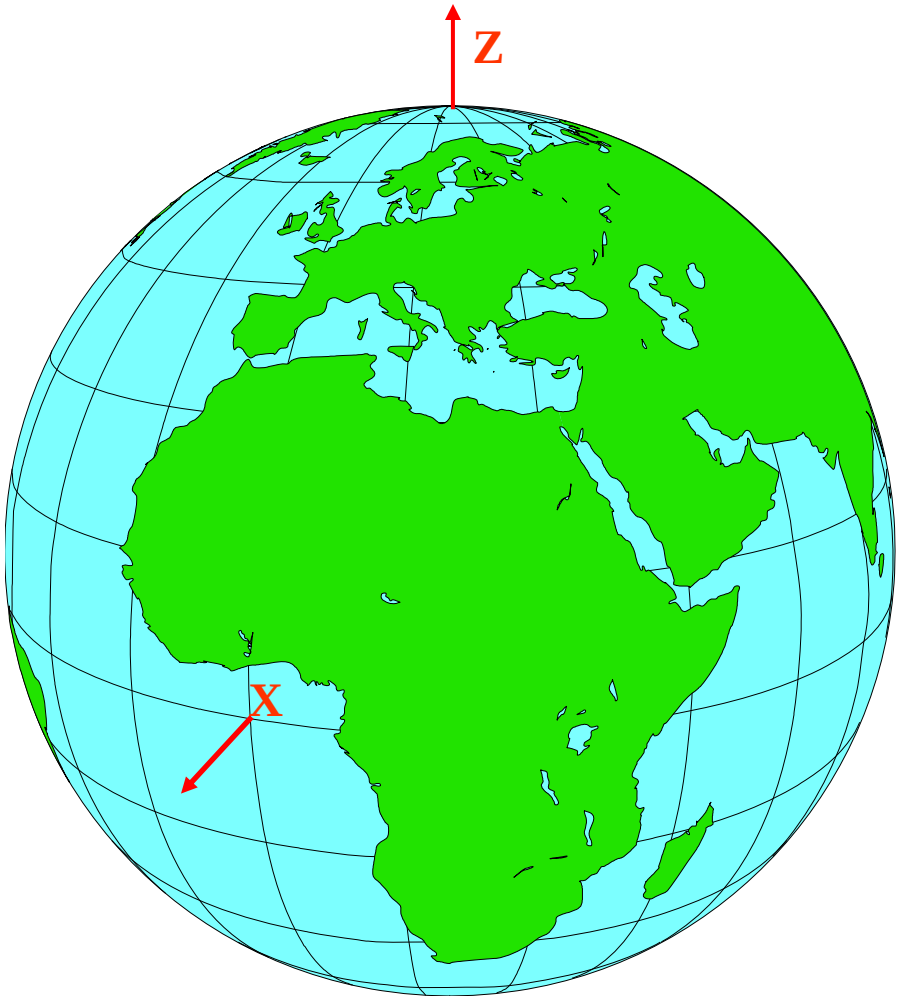
RZ (NWL9D->WGS72) = 0.260” (pv)
RZ (WGS72->WGS84) = 0.554” (pv)

Greenwich Astronomic Meridian
E 000° 00’ 00.554” ?

Geodetic Interpretation of the 7 Parameters w.r.t. WGS84

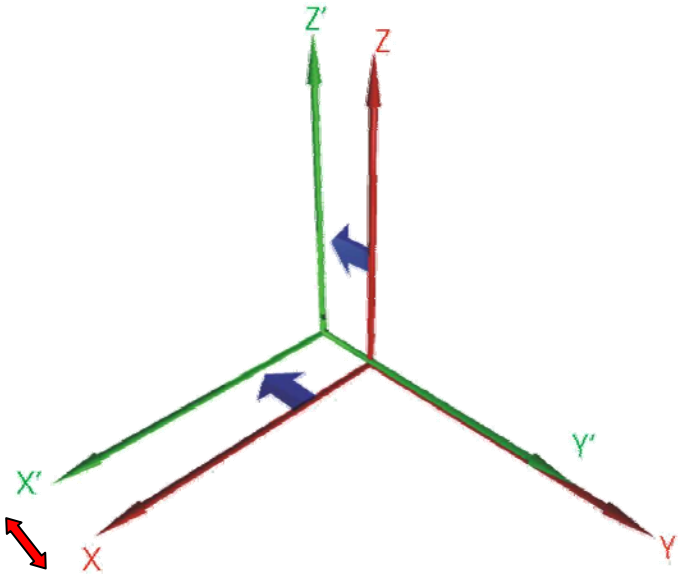
- **RZ** - change in longitude w.r.t. BIH Zero Meridian
- **ΔX, ΔY, ΔZ** - lateral translations at the geocenter
- **ΔS** - change in scale of linear unit w.r.t. VLBI
- **RX, RY** - your guess! Changes horizontal orientation w.r.t. CIO north. The effects of these parameters are location-specific.

X-Axis Near West Africa

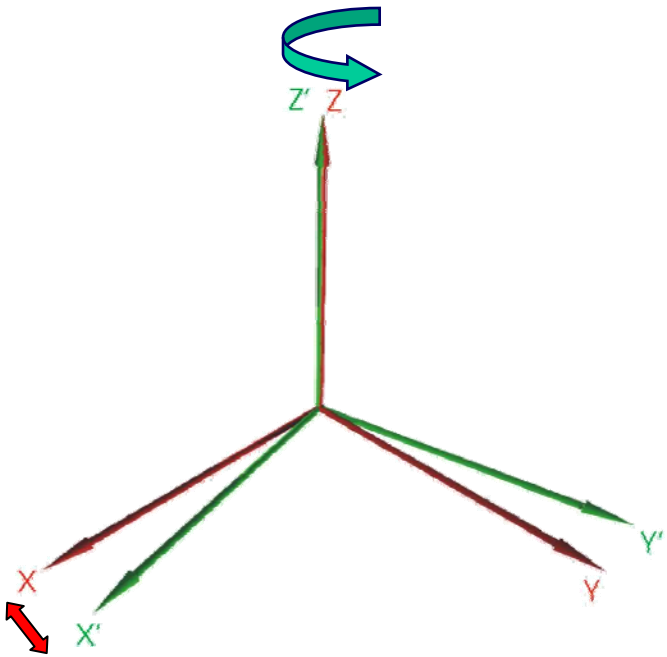


Longitude Correlation - West Africa

ΔY

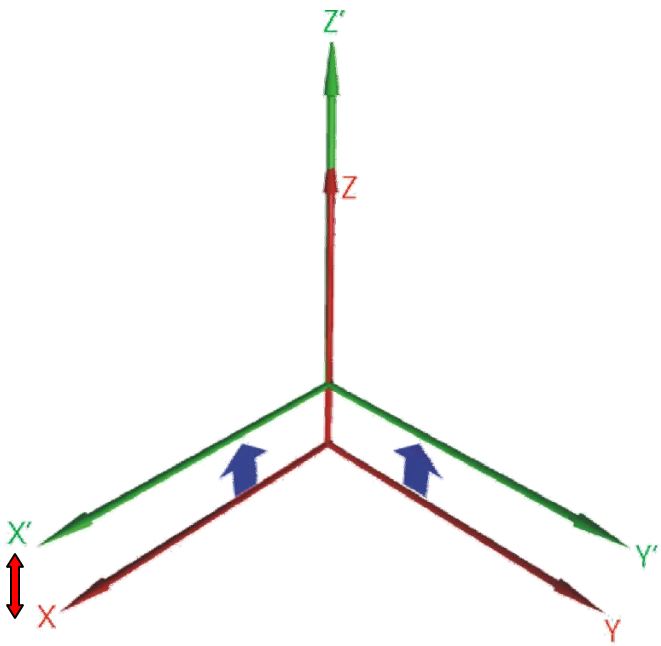


RZ

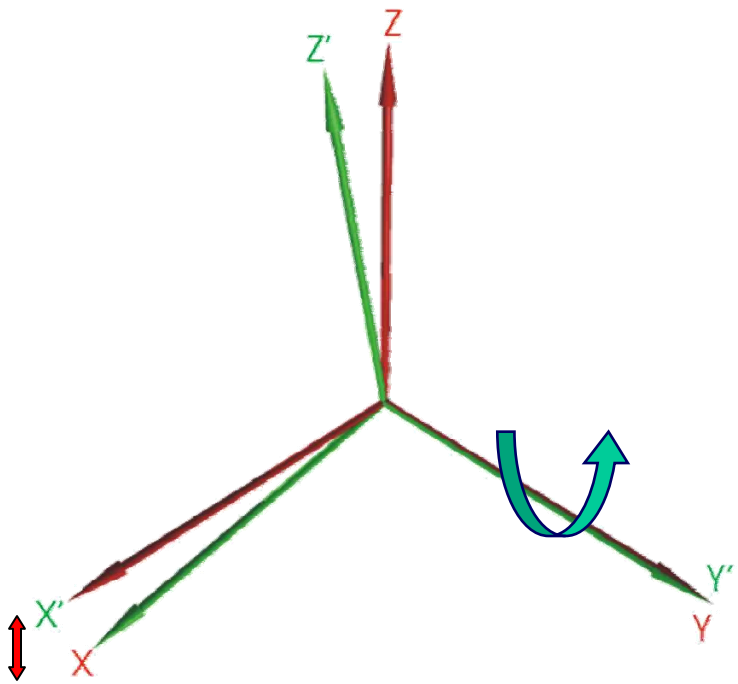


Latitude Correlation - West Africa

ΔZ

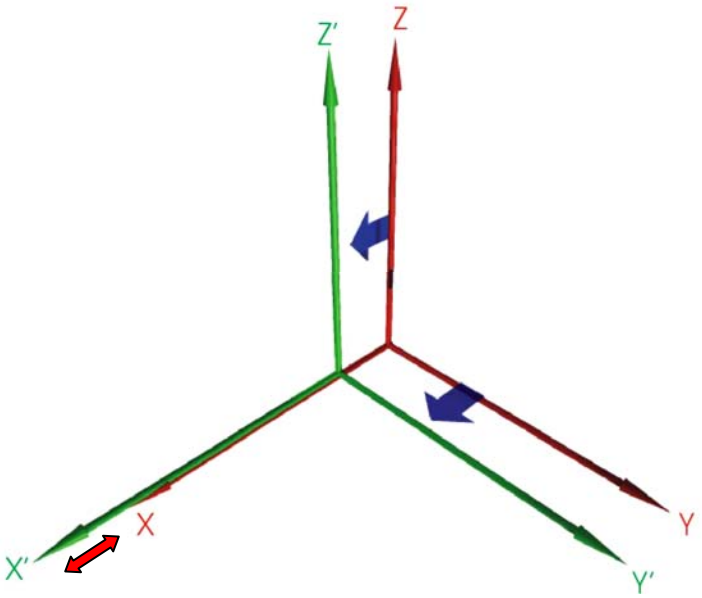


RY

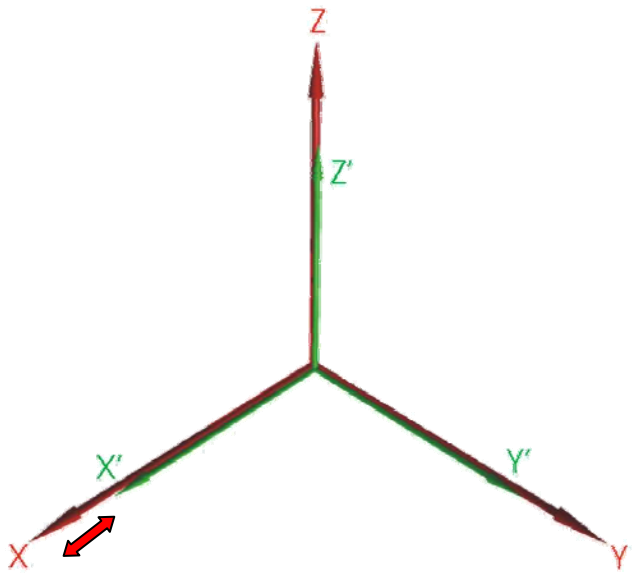


Height Correlation - West Africa

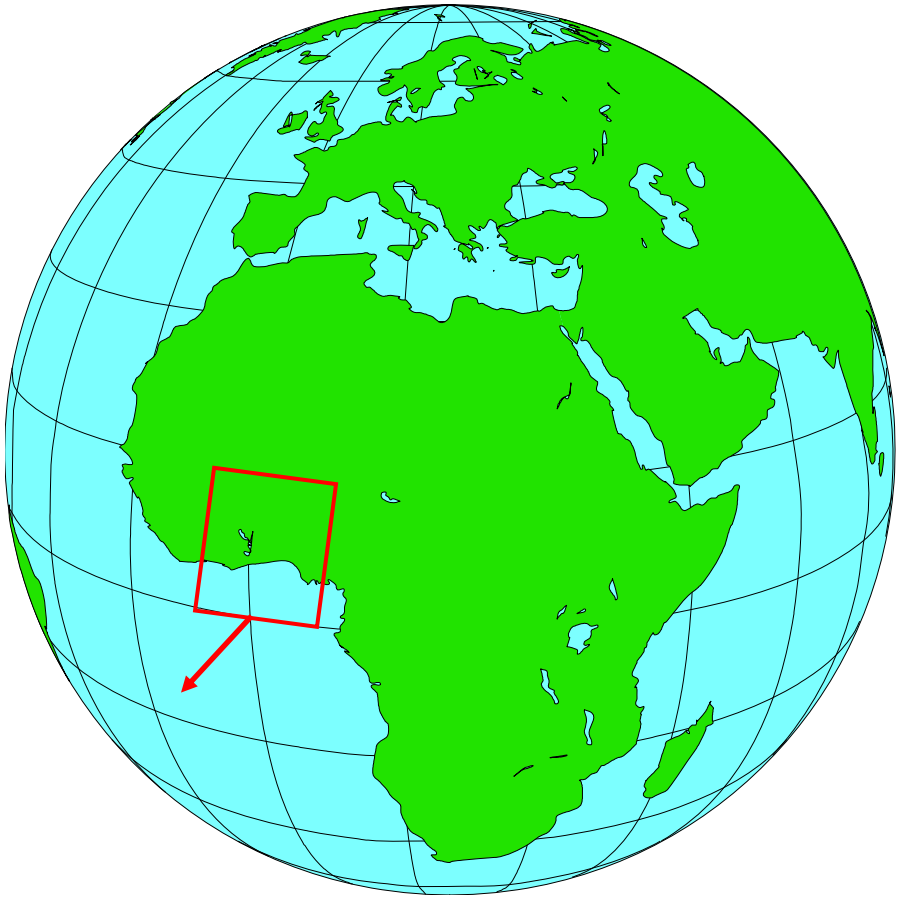
ΔX



ΔS

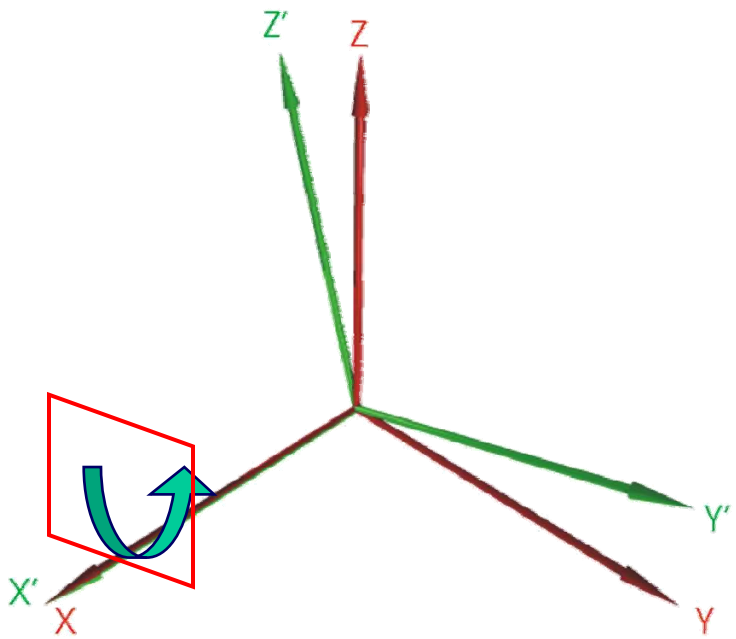


North Orientation - West Africa

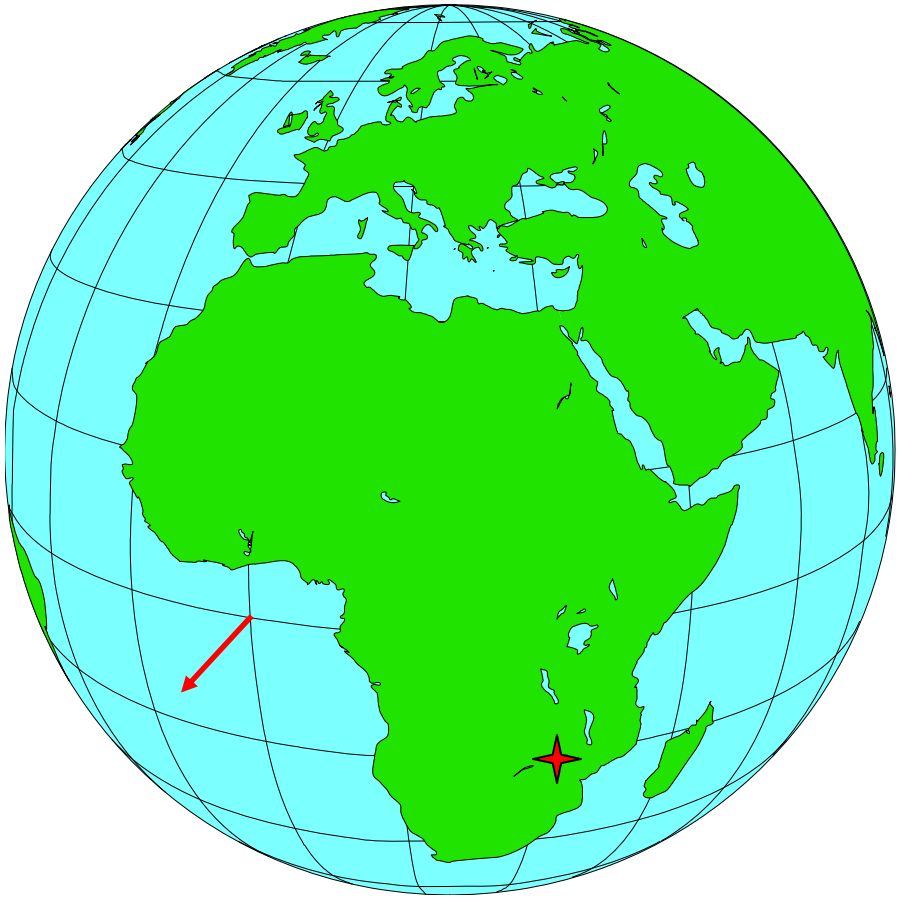


North Orientation - West Africa

RX



Harare, Zimbabwe

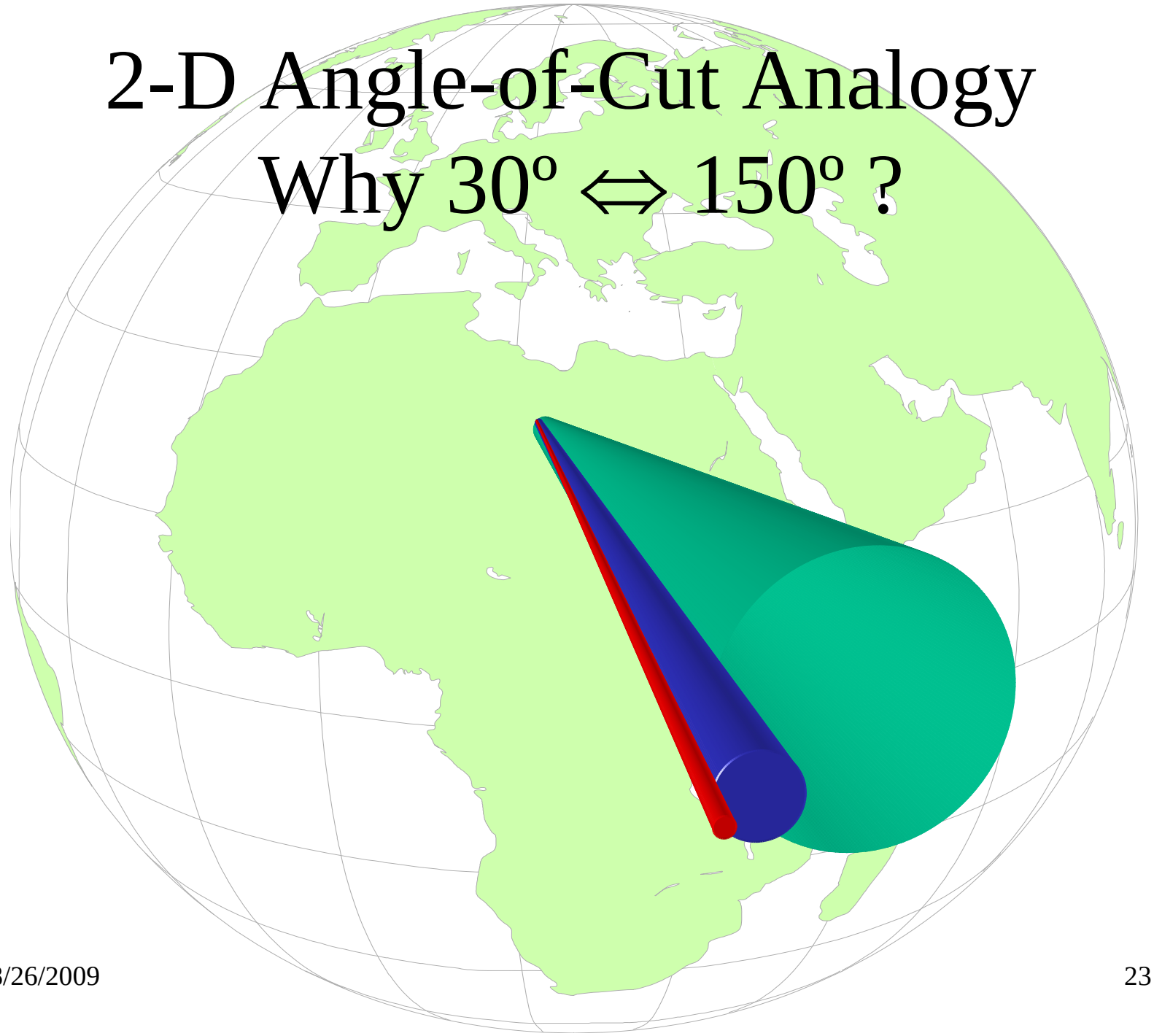


Five 3-Parameter Shifts at Harare

ARC 1950 | S 28-00-00.0000
E 31-00-00.0000
0.0000m | to | S 28-00-01.6119
E 30-59-59.8721
24.1673m | WGS84

	DMA	#2	#3	#4	#5
• ΔXm	• -143	• 0	• 0	• 0	• 0
• ΔYm	• - 90	• 0	• 0	• 0	• -26.540
• ΔZm	• -294	• 0	• 0	• 0	• 0
• RX”pv	• 0	• 21.2927	• -5.8558	• 0	• 0
• RY”pv	• 0	• 0	• 9.2754	• 12.7939	• 12.5529
• RZ”pv	• 0	• -9.5105	• 0	• -3.6077	• -2.7095
• ΔSppm	• 0	• 1.8965	• 1.8965	• 1.8965	• 0

2-D Angle-of-Cut Analogy
Why 30º ⇔ 150º ?



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NGA Molodensky Model - 1

http://earth-info.nima.mil/GandG/datums/wgsdt.html

Seven-Parameter geometric transformation

MOLODENSKY Model

The transformation is between a non-global local datum and a geocentric global geodetic system. The rotations are to be considered about the three axes at the "initial" point of the local datum. The scale factor is also considered with respect to the initial point.

NGA Molodensky Model - 2

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_i = \begin{bmatrix} U \\ V \\ W \end{bmatrix} + \begin{bmatrix} \Delta U \\ \Delta V \\ \Delta W \end{bmatrix} + \begin{bmatrix} 0 & \varpi & -\psi \\ -\varpi & 0 & \varepsilon \\ \psi & -\varepsilon & 0 \end{bmatrix} \begin{bmatrix} U - U' \\ V - V' \\ W - W' \end{bmatrix}_i + \Delta S \begin{bmatrix} U - U' \\ V - V' \\ W - W' \end{bmatrix}_i$$

where "i" denotes any point common to the local datum and geodetic system and the (U',V',W') are the coordinates of the "initial" point of the local datum. The three angles correspond to the small rotations taken positive in the counter clock-wise mode, when viewed from the end of the respective axes (at the "initial" point) towards the origin.

Note: When the "initial" point of the local datum (U',V',W') is not provided, assume values of (0,0,0).

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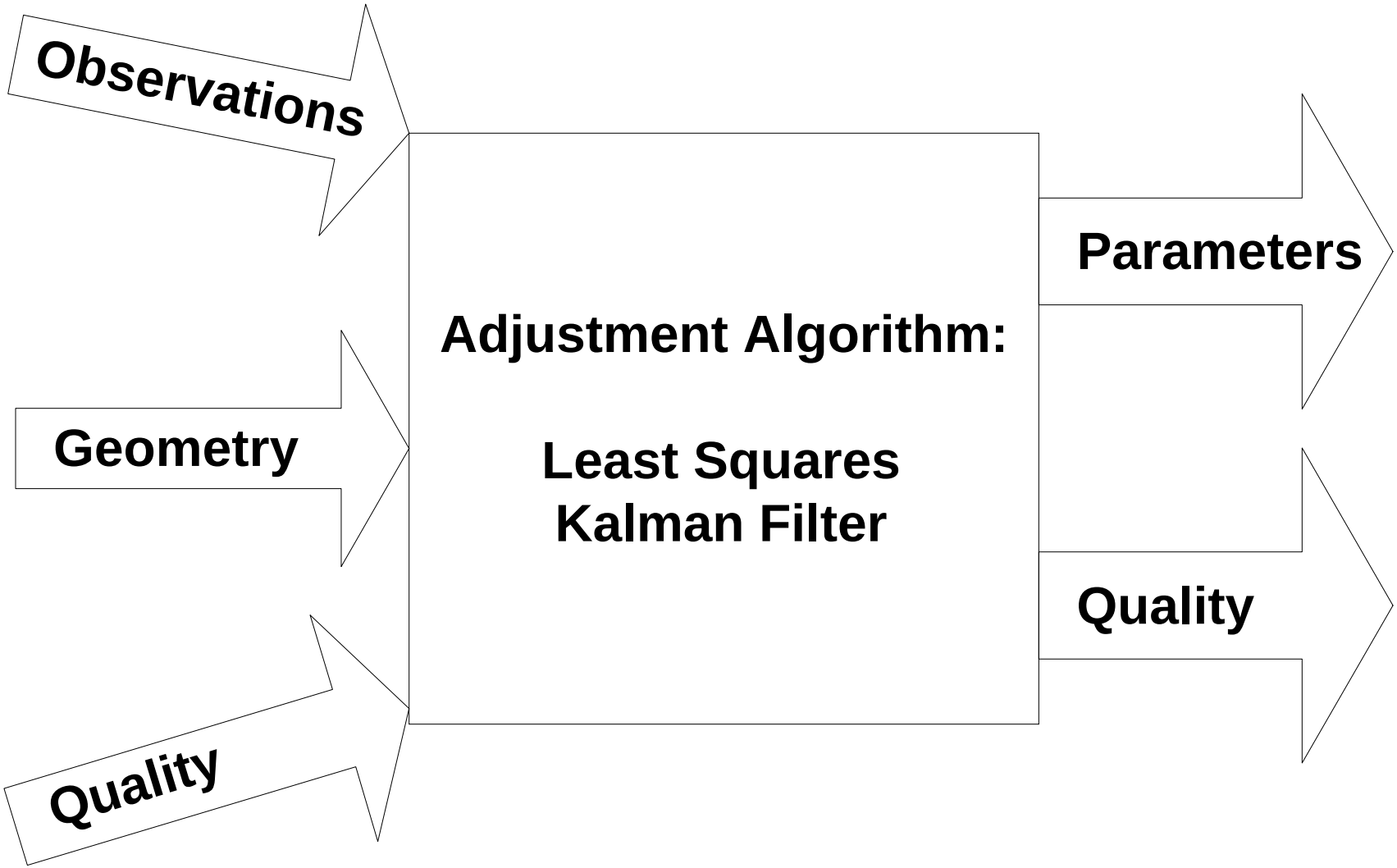
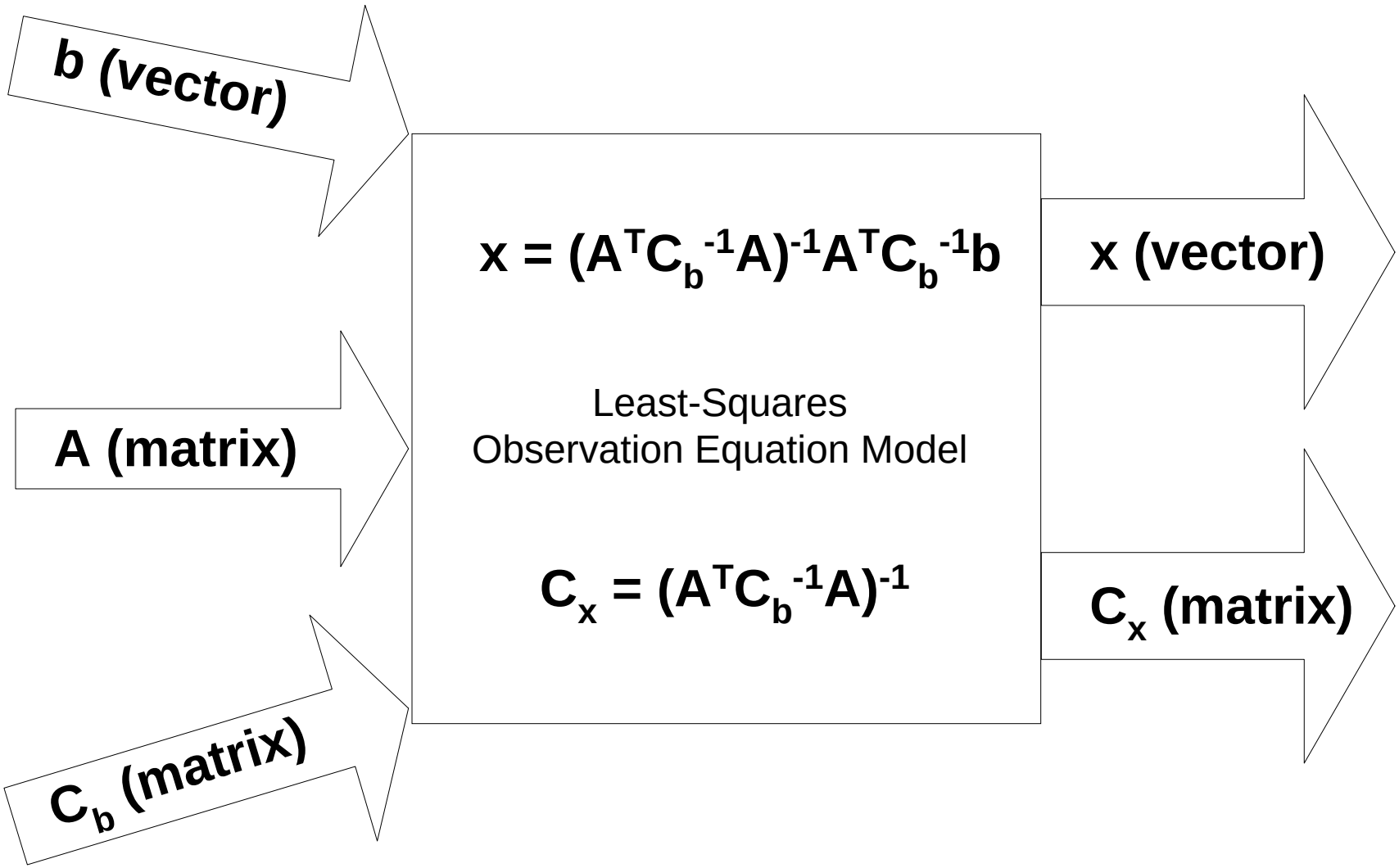
Adjustment Inputs and Outputs



Adjustment Inputs and Outputs



Snippet of Derivation Code

```
% Populate the design matrix (A) and the observations vector (b)
for this = 1:pts
   A(this*3-2, :) = [1  0  0  0          -Zfrom(this) Yfrom(this) Xfrom(this)];
   A(this*3-1, :) = [0  1  0  Zfrom(this) 0          -Xfrom(this) Yfrom(this)];
   A(this*3,   :) = [0  0  1 -Yfrom(this) Xfrom(this) 0           Zfrom(this)];

   b(this*3-2) =  Xto(this) - Xfrom(this);
   b(this*3-1) =  Yto(this) - Yfrom(this);
   b(this*3)   =  Zto(this) - Zfrom(this);
end

% Define covariance matrix of the observations.  Assume 1-m SD error per axis.
Cb = eye(pts*3);

% Compute covariance matrix of the parameters.
Cx = inv(A’*inv(Cb)*A);

% Solve for the 7 parameters.  Rotations are in radians.
x = Cx*A'*b

% Solve for post-adjustment residuals.
b2 = A*x-b;

% Solve for variance factor
vf = b2'*b2/(pts*3-length(x));

% Scale Cx by vf, solve for SDs of parameters and the correlation matrix
SD = (diag(Cx*vf)).^.5;
correlation = cor(Cx)
```

Brief Review of Correlation

Normalizing the Covariance Matrix (Cx)

$$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j}$$

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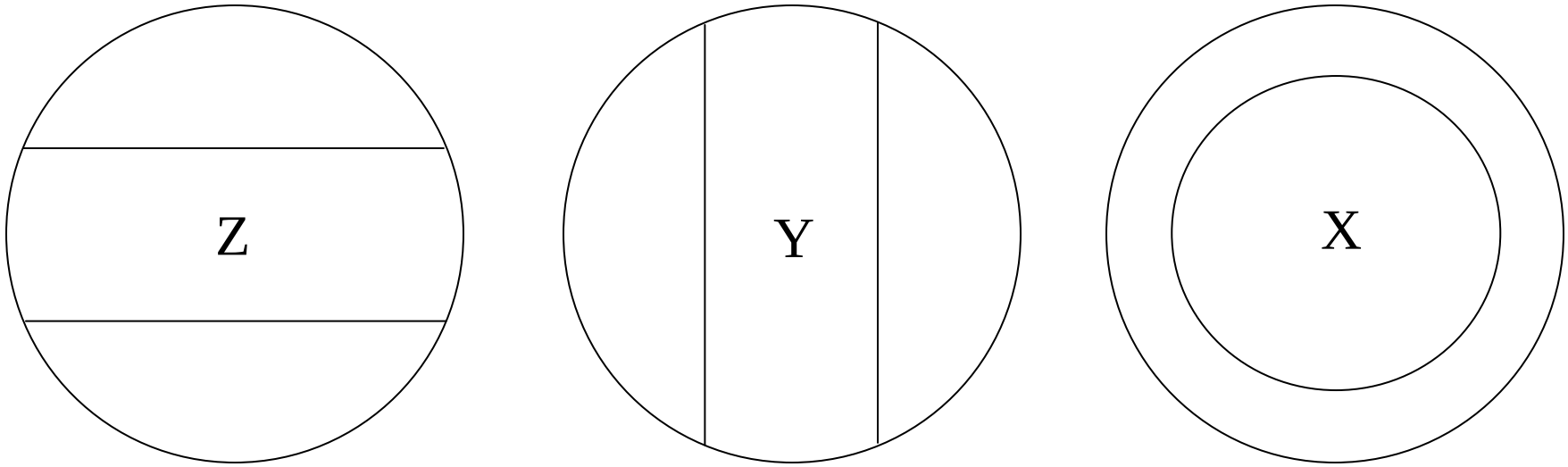
Distribution of 1M Random Points

Ellipsoid Thirds Along Each Axis



332688	333336	333758
335854	333009	332650
331458	333655	333592

Global and Continental Areas

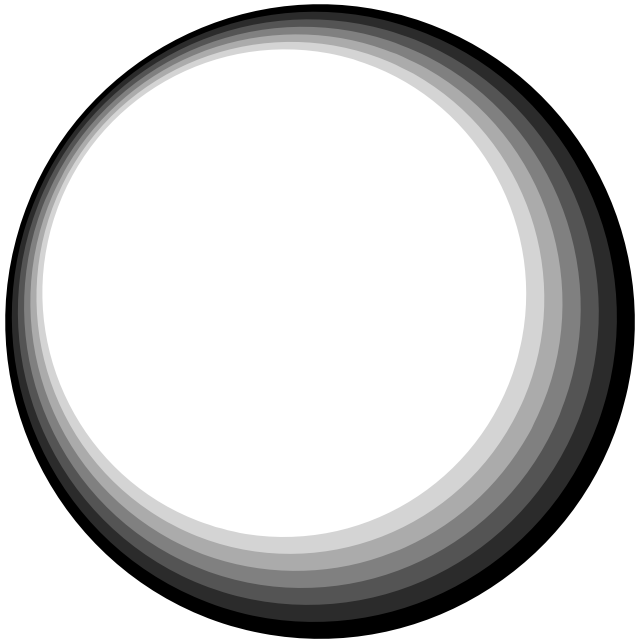
Induced Datum Shift and *a priori* Errors

• **ΔXm**	• **+700m**
• **ΔYm**	• **-500m**
• **ΔZm**	• **+200m**
• **RX”**	• **-3”**
• **RY”**	• **+5”**
• **RZ”**	• **-2”**
• **ΔSppm**	• **+3ppm**

- **X ±1m 1σ**
- **Y ±1m 1σ**
- **Z ±1m 1σ**

Entire World: Monte Carlo

19 random points, 1-m SD / axis, 7 parameters as defined



RMS = 0.98, vf = 1.09, sduw = 1.04

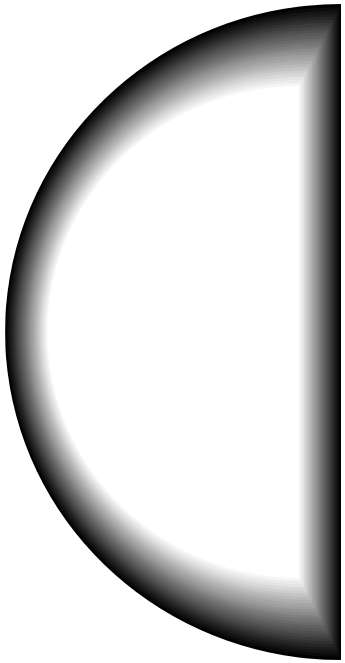
	7P	SD	SDsc
ΔX	699.674	0.233	0.243
ΔY	-500.062	0.232	0.242
ΔZ	199.817	0.233	0.243
RX	-2.992	0.010	0.010
RY	5.006	0.009	0.009
RZ	-1.997	0.009	0.010
ΔS	3.031	0.036	0.038

Correlation_Matrix

1.00	0.00	0.00	-0.02	0.09	0.17	0.02
0.00	1.00	0.01	-0.08	0.01	0.03	-0.13
0.00	0.01	1.00	-0.17	-0.01	0.02	0.06
-0.02	-0.08	-0.17	1.00	-0.05	-0.11	0.00
0.09	0.01	-0.01	-0.05	1.00	0.13	-0.00
0.17	0.03	0.02	-0.11	0.13	1.00	-0.00
0.02	-0.13	0.06	-0.00	0.00	0.00	1.00

Hemisphere: Monte Carlo

19 random points, 1-m SD / axis, 7 parameters as defined



RMS = 0.84, vf = 0.80, sduw = 0.90

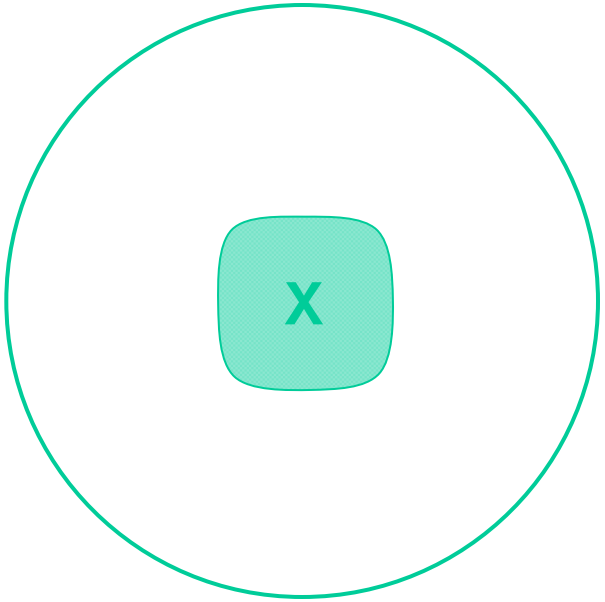
	7P	SD	SDsc
ΔX	699.647	0.272	0.244
ΔY	-499.741	0.294	0.263
ΔZ	199.523	0.302	0.271
RX	-3.020	0.009	0.008
RY	5.000	0.012	0.011
RZ	-2.007	0.011	0.010
ΔS	3.020	0.042	0.038

Correlation_Matrix

1.00	-0.01	0.07	0.00	0.21	0.02	-0.49
-0.01	1.00	-0.05	-0.19	-0.07	-0.61	0.00
0.07	-0.05	1.00	0.02	0.64	0.08	0.13
0.00	-0.19	0.02	1.00	0.02	0.08	-0.00
0.21	-0.07	0.64	0.02	1.00	0.12	-0.00
0.02	-0.61	0.08	0.08	0.12	1.00	0.00
-0.49	0.00	0.13	-0.00	-0.00	-0.00	1.00

“Australia-Sized”: Monte Carlo

19 random points, 1m SD / axis, 7 parameters as defined



RMS = 0.85, vf = 0.83, sduw = 0.91

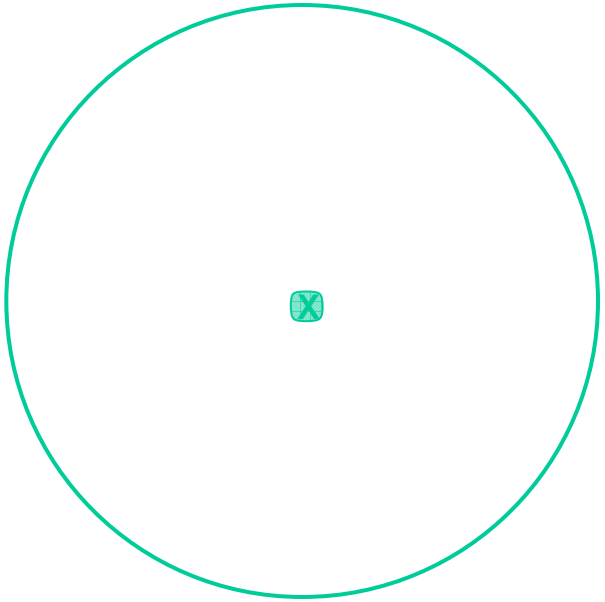
	7P	SD	SDsc
ΔX	698.919	1.202	1.095
ΔY	-501.506	1.696	1.544
ΔZ	201.359	1.714	1.561
RX	-2.977	0.039	0.035
RY	5.052	0.056	0.051
RZ	-1.946	0.055	0.050
Δ	3.149	0.188	0.172

Correlation_Matrix

1.00	0.01	-0.02	-0.00	-0.04	-0.01	-0.98
0.01	1.00	-0.17	0.01	-0.17	-0.99	0.00
-0.02	-0.17	1.00	0.03	0.99	0.17	-0.02
-0.00	0.01	0.03	1.00	0.03	0.01	0.00
-0.04	-0.17	0.99	0.03	1.00	0.17	0.00
-0.01	-0.99	0.17	0.01	0.17	1.00	-0.00
-0.98	0.00	-0.02	-0.00	-0.00	-0.00	1.00

“Cyprus-Sized”: Monte Carlo

19 random points, 1m SD / axis, 7 parameters as defined



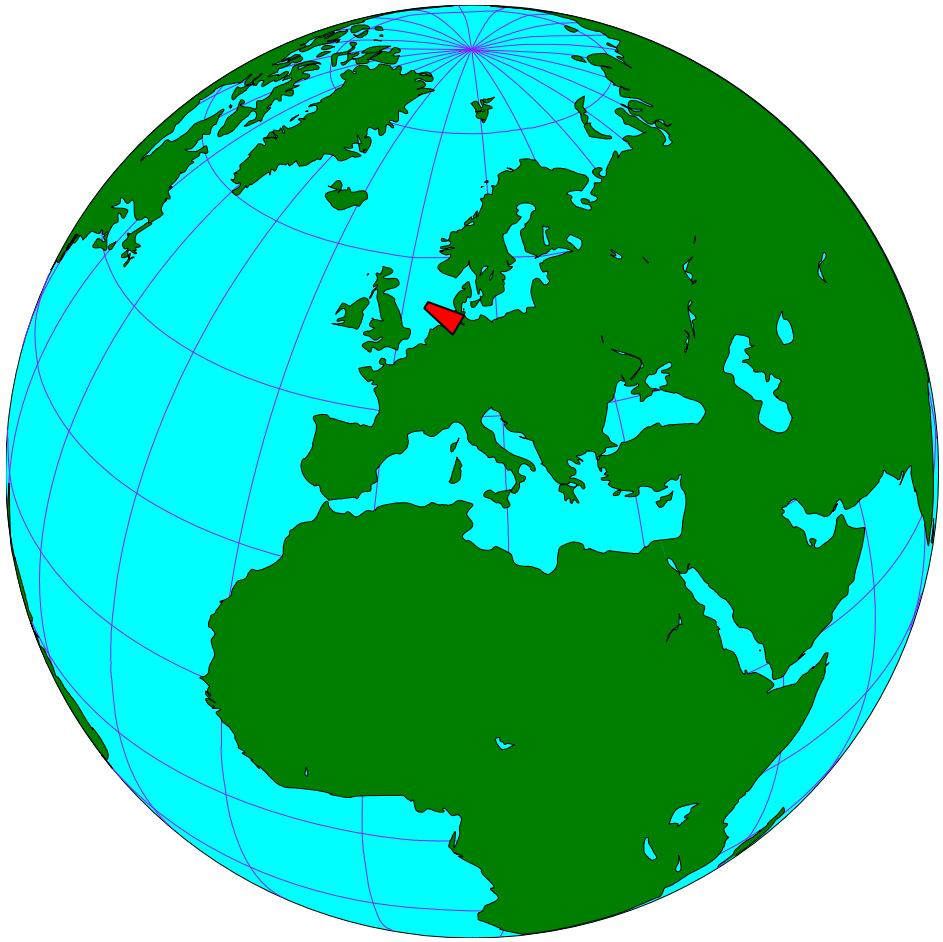
RMS = 0.97, vf = 1.08, sduw = 1.04

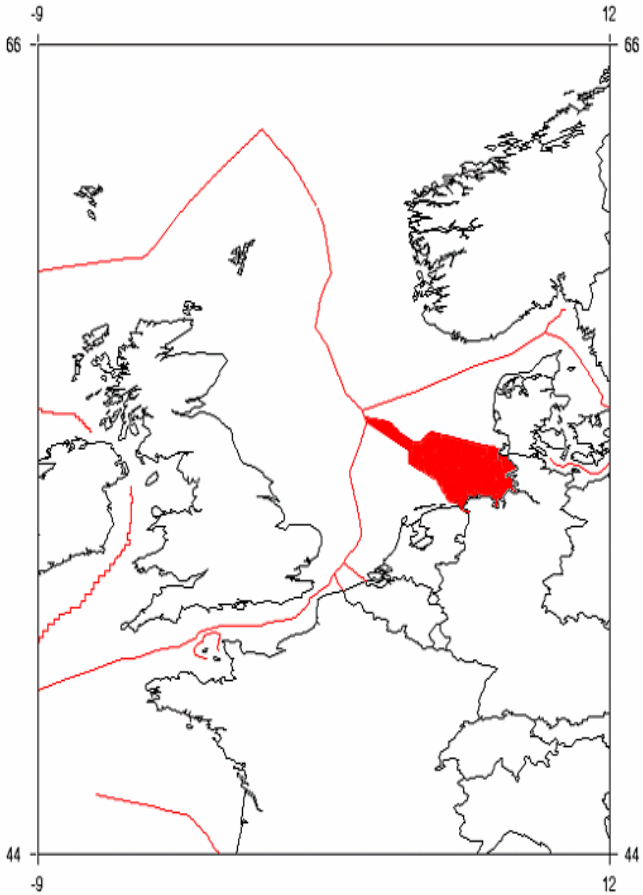
	7P	SD	SDsc
ΔX	761.639	31.764	33.019
ΔY	-561.148	49.999	51.974
ΔZ	96.197	61.947	64.394
RX	-3.374	1.027	1.068
RY	1.631	2.003	2.082
RZ	-0.027	1.617	1.681
ΔS	-6.650	4.980	5.177

Correlation_Matrix

1.00	-0.00	-0.00	-0.00	-0.00	-0.00	-1.00
-0.00	1.00	0.58	0.00	0.58	-1.00	0.00
-0.00	0.58	1.00	0.00	1.00	-0.58	-0.00
-0.00	0.00	0.00	1.00	0.00	-0.00	0.00
-0.00	0.58	1.00	0.00	1.00	-0.58	0.00
-0.00	-1.00	-0.58	-0.00	-0.58	1.00	-0.00
-1.00	0.00	-0.00	-0.00	0.00	0.00	1.00

German North Sea





Proposed German No. Sea 7-P Shift

Geodetic Transformations (Single-Step)

EPSG geodesy parameters

***Search Criteria:* Germany - offshore North Sea.**

#2 ***Transformation Code:*** ***Name:*** **ED50 to WGS 84**

Source CRS: code = **4230** ***name =*** **ED50**

Source Ellipsoid:: **International 1924**

Semi-major axis (a) = **6378388**

Semi-minor axis (b) = ***inverse flattening =*** **297**

Target CRS: code = **4326** ***name =*** **WGS 84**

Target Ellipsoid:: **WGS 84**

Semi-major axis (a) = **6378137 metre**

Semi-minor axis (b) = ***inverse flattening =*** **298.257223563**

Data Source: **EPSG** ***Change ID:*** **Rev. Date:** **31-Dec-03**

Information Source:

Area of Use: **Germany - offshore North Sea.**

Scope: **Recommended transformation for Germany North Sea petroleum purposes.**

Remarks: **Approximation to better than 0.5m of transformation adopted in June 2003.**

Proposed German No. Sea 7-P Shift

Trf Variant: ***Version:*** **EPSG-Ger Nsea**

Transformation method: **Position Vector 7-param. transformation**

For Polynomial transformation methods only:

Unit Source Offsets: ***Unit Target Offsets:***

Transformation Parameter Name	***Value***	***Unit of Measure***
X-axis translation	***-157.89***	***metre***
Y-axis translation	***-17.16***	***metre***
Z-axis translation	***-78.41***	***metre***
X-axis rotation	***2.118***	***arc-second***
Y-axis rotation	***2.697***	***arc-second***
Z-axis rotation	***-1.434***	***arc-second***
Scale difference	***-5.38***	***parts per million***

Area of Use: Germany - offshore North Sea.

Scope: Recommended transformation for Germany North Sea petroleum purposes.

Remarks: Approximation to better than 0.5m of transformation adopted in June 2003 .

German North Sea

Induced Datum Shift and *a priori* Errors

- **ΔXm**
- **ΔYm**
- **ΔZm**
- **RX”**
- **RY”**
- **RZ”**
- **ΔSppm**

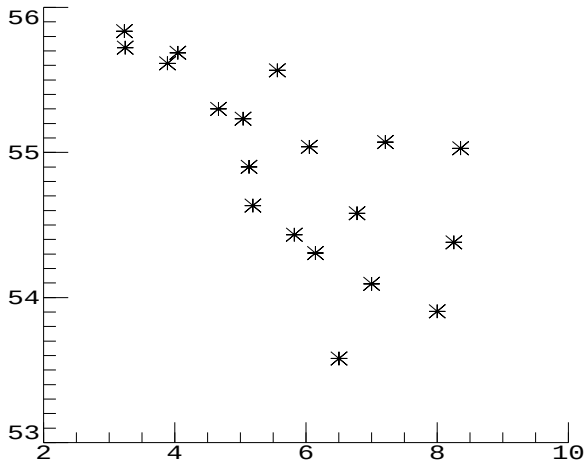
- **0m**
- **0m**
- **0m**
- **0”**
- **0”**
- **0”**
- **0ppm**

- **X ±1m 1σ**
- **Y ±1m 1σ**
- **Z ±1m 1σ**

German North Sea: Monte Carlo 1

19 random points, 1-m SD / axis, all 7 parameters = 0

Helmert with 7 Parameters



RMS = 0.79, vf = 0.71, sduw = 0.84

	7P	SD	SDsc
ΔX	-19.379	25.595	21.549
ΔY	10.495	19.159	16.131
ΔZ	-0.541	19.138	16.113
RX	0.155	0.537	0.452
RY	0.474	0.959	0.807
RZ	-0.425	0.501	0.422
ΔS	1.705	1.884	1.586

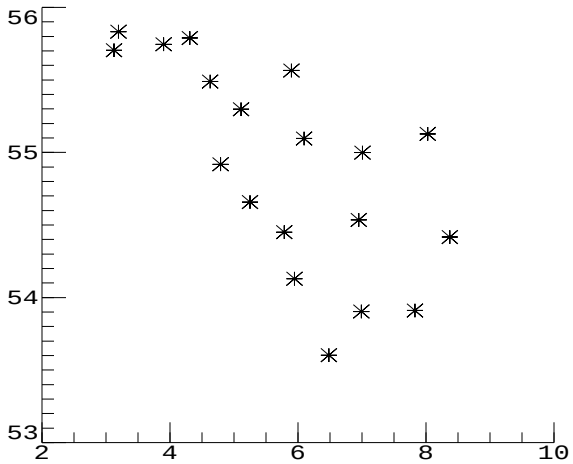
Correlation_Matrix

1.00	-0.65	-0.69	-0.57	-0.96	0.55	-0.27
-0.65	1.00	0.56	0.91	0.67	-0.77	-0.04
-0.69	0.56	1.00	0.47	0.86	-0.47	-0.51
-0.57	0.91	0.47	1.00	0.58	-0.44	0.00
-0.96	0.67	0.86	0.58	1.00	-0.55	0.00
0.55	-0.77	-0.47	-0.44	-0.55	1.00	-0.00
-0.27	-0.04	-0.51	-0.00	-0.00	0.00	1.00

German North Sea: Monte Carlo 2

19 random points, 1-m SD / axis, all 7 parameters = 0

Helmert with 7 Parameters



RMS = 0.97, vf = 1.07, sduw = 1.03

	7P	SD	SDsc
ΔX	18.504	23.623	24.388
ΔY	-34.929	19.208	19.830
ΔZ	-11.071	17.874	18.453
RX	-0.955	0.539	0.557
RY	-0.693	0.879	0.907
RZ	0.638	0.497	0.513
ΔS	0.106	1.862	1.922

Correlation_Matrix

[1.00	-0.63	-0.65	-0.56	-0.96	0.54	-0.29]
[-0.63	1.00	0.54	0.91	0.66	-0.77	-0.04]
[-0.65	0.54	1.00	0.45	0.84	-0.44	-0.54]
[-0.56	0.91	0.45	1.00	0.58	-0.44	-0.00]
[-0.96	0.66	0.84	0.58	1.00	-0.54	-0.00]
[0.54	-0.77	-0.44	-0.44	-0.54	1.00	0.00]
[-0.29	-0.04	-0.54	-0.00	-0.00	0.00	1.00]

Topics

- Statement of the issues
- Geographicals to Cartesians to Geographicals
- Heuristic explanation of correlation problem
- The Molodensky Model at the geocenter (Helmert)
- The derivation of a 7-parameter shift
- Monte Carlo correlation w.r.t. datum area
- **The Molodensky Model at the surface (M-B) solves correlation problem**
- Dilution of Precision (P7DOP)
- Conclusion
- Reversibility in Molodensky-Badekas (Appendix)

NGA Molodensky Model - 2

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_i = \begin{bmatrix} U \\ V \\ W \end{bmatrix} + \begin{bmatrix} \Delta U \\ \Delta V \\ \Delta W \end{bmatrix} + \begin{bmatrix} 0 & \varpi & -\psi \\ -\varpi & 0 & \varepsilon \\ \psi & -\varepsilon & 0 \end{bmatrix} \begin{bmatrix} U - U' \\ V - V' \\ W - W' \end{bmatrix}_i + \Delta S \begin{bmatrix} U - U' \\ V - V' \\ W - W' \end{bmatrix}_i$$

where "i" denotes any point common to the local datum and geodetic system and the (U',V',W') are the coordinates of the "initial" point of the local datum. The three angles correspond to the small rotations taken positive in the counter clock-wise mode, when viewed from the end of the respective axes (at the "initial" point) towards the origin.

Note: When the "initial" point of the local datum (U',V',W') is not provided, assume values of (0,0,0).

German North Sea Molodensky-Badekas at Barycenter

Induced Datum Shift and *a priori* Errors

- **ΔXm**
- **ΔYm**
- **ΔZm**
- **RX”**
- **RY”**
- **RZ”**
- **ΔSppm**

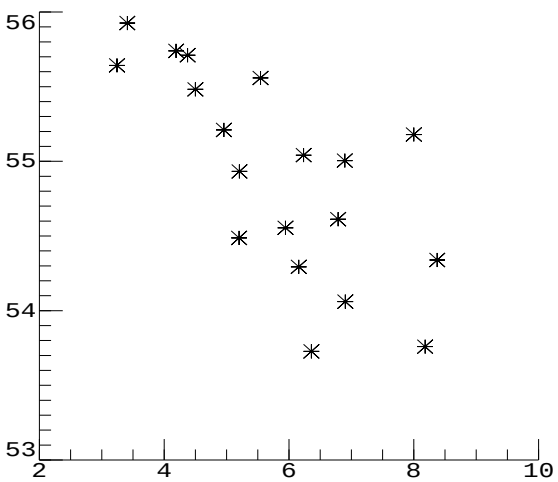
- **0m**
- **0m**
- **0m**
- **0”**
- **0”**
- **0”**
- **0ppm**

- **X ±1m 1σ**
- **Y ±1m 1σ**
- **Z ±1m 1σ**

German North Sea: Monte Carlo 3

19 random points, 1-m SD / axis, all 7 parameters = 0

Barycenter Molodensky-Badekas with 7 Parameters



Xc=3654621.110, Yc=373231.167, Zc=5195307.741

RMS = 0.81, vf = 0.74, sduw = 0.86

	7P	SD	SDsc
ΔX	-0.130	0.229	0.198
ΔY	0.070	0.229	0.198
ΔZ	-0.025	0.229	0.198
RX	-0.794	0.570	0.491
RY	-1.747	0.953	0.821
RZ	0.674	0.525	0.452
ΔS	-1.106	1.914	1.649

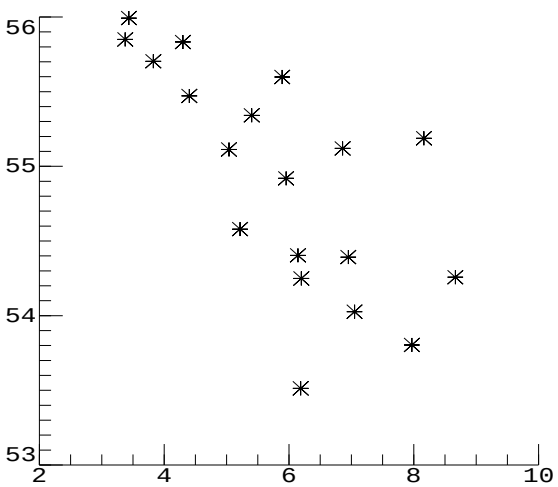
Correlation_Matrix

1.00	0.00	0.00	0.00	0.00	-0.00	-0.00
0.00	1.00	0.00	0.00	0.00	-0.00	0.00
0.00	0.00	1.00	0.00	0.00	-0.00	0.00
0.00	0.00	0.00	1.00	0.61	-0.47	0.00
0.00	0.00	0.00	0.61	1.00	-0.57	0.00
-0.00	-0.00	-0.00	-0.47	-0.57	1.00	0.00
-0.00	0.00	-0.00	-0.00	-0.00	0.00	1.00

German North Sea: Monte Carlo 4

19 random points, 1-m SD / axis, all 7 parameters = 0

Barycenter Molodensky-Badekas with 7 Parameters



Xc=3653872.647, Yc=375185.984, Zc=5195534.393

RMS = 0.96, vf = 1.06, sduw = 1.03

	7P	SD	SDsc
ΔX	-0.199	0.229	0.236
ΔY	-0.091	0.229	0.236
ΔZ	-0.188	0.229	0.236
RX	0.163	0.546	0.562
RY	0.455	0.847	0.873
RZ	0.062	0.498	0.513
ΔS	3.679	1.814	1.868

Correlation_Matrix

1.00	-0.00	-0.00	-0.00	-0.00	0.00	-0.00
-0.00	1.00	0.00	0.00	0.00	-0.00	-0.00
-0.00	0.00	1.00	0.00	0.00	-0.00	-0.00
-0.00	0.00	0.00	1.00	0.60	-0.48	-0.00
-0.00	0.00	0.00	0.60	1.00	-0.56	-0.00
0.00	-0.00	-0.00	-0.48	-0.56	1.00	0.00
-0.00	-0.00	-0.00	-0.00	-0.00	0.00	1.00

Proposed German No. Sea 7-P Shift

Trf Variant: ***Version:*** **EPSG-Ger Nsea**

Transformation method: **Position Vector 7-param. transformation**

For Polynomial transformation methods only:

Unit Source Offsets: ***Unit Target Offsets:***

Transformation Parameter Name	***Value***	***Unit of Measure***
X-axis translation	***-157.89***	***metre***
Y-axis translation	***-17.16***	***metre***
Z-axis translation	***-78.41***	***metre***
X-axis rotation	***2.118***	***arc-second***
Y-axis rotation	***2.697***	***arc-second***
Z-axis rotation	***-1.434***	***arc-second***
Scale difference	***-5.38***	***parts per million***

Area of Use: Germany - offshore North Sea.

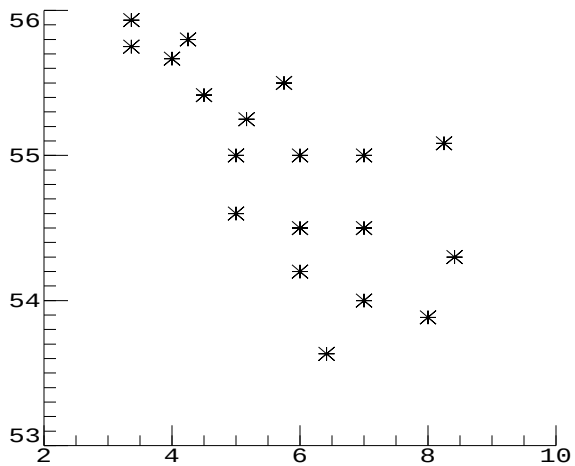
Scope: ***Recommended transformation for Germany North Sea petroleum purposes.***

Remarks: ***Approximation to better than 0.5m of transformation adopted in June 2003.***

German North Sea: MRE Match

19 MRE points, no induced random error

Helmert with 7 Parameters



RMS = 0.118, vf = 0.016, sduw = 0.126

	7P	SD	SDsc
ΔX	-157.893	24.889	3.145
ΔY	-17.164	19.882	2.512
ΔZ	-78.412	18.614	2.352
RX	2.118	0.554	0.070
RY	2.697	0.929	0.117
RZ	-1.434	0.511	0.065
ΔS	-5.380	1.871	0.236

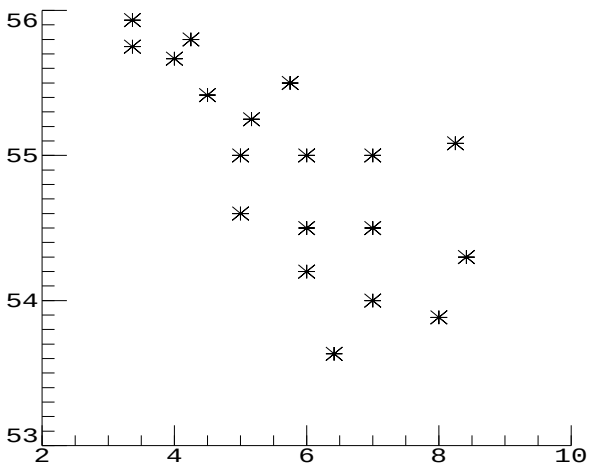
Correlation_Matrix

1.00	-0.66	-0.67	-0.59	-0.96	0.57	-0.27
-0.66	1.00	0.57	0.91	0.68	-0.78	-0.04
-0.67	0.57	1.00	0.48	0.85	-0.48	-0.52
-0.59	0.91	0.48	1.00	0.61	-0.47	0.00
-0.96	0.68	0.85	0.61	1.00	-0.57	0.00
0.57	-0.78	-0.48	-0.47	-0.57	1.00	-0.00
-0.27	-0.04	-0.52	0.00	0.00	-0.00	1.00

German North Sea: MRE Match

19 MRE points, no induced random error

Barycenter Molodensky-Badekas with 7 Parameters



Xc=3655727.054, Yc=373465.142, Zc=5194453.8

RMS = 0.118, vf = 0.016, sduw = 0.126

	7P	SD	SDsc
ΔX	-107.055	0.229	0.029
ΔY	-97.928	0.229	0.029
ΔZ	-150.318	0.229	0.029
RX	2.118	0.554	0.070
RY	2.697	0.929	0.117
RZ	-1.434	0.511	0.065
ΔS	-5.380	1.871	0.236

Correlation_Matrix

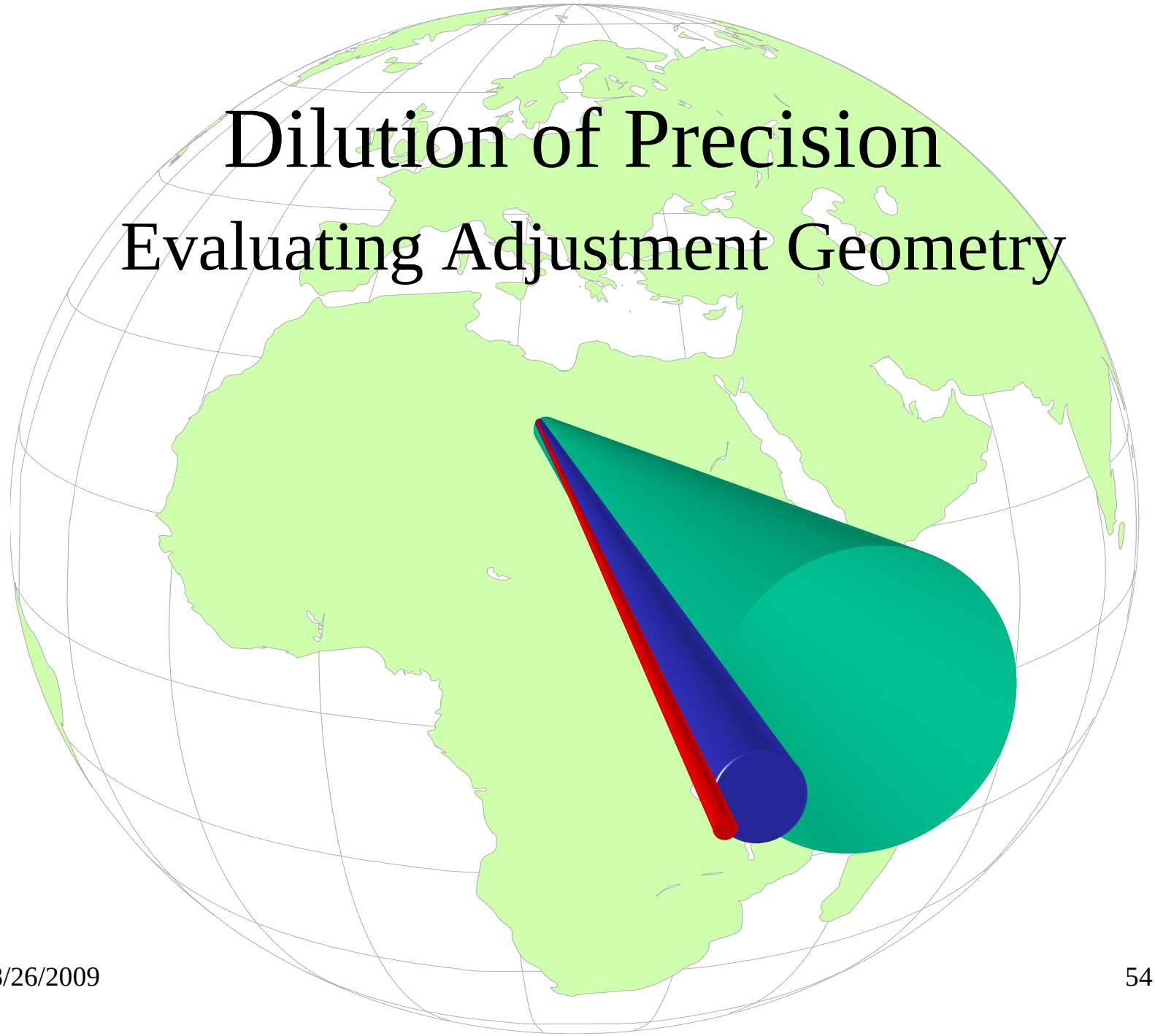
1.00	-0.00	-0.00	-0.00	-0.00	0.00	-0.00
-0.00	1.00	0.00	0.00	0.00	-0.00	-0.00
-0.00	0.00	1.00	0.00	0.00	-0.00	-0.00
-0.00	0.00	0.00	1.00	0.61	-0.47	-0.00
-0.00	0.00	0.00	0.61	1.00	-0.57	-0.00
0.00	-0.00	-0.00	-0.47	-0.57	1.00	0.00
-0.00	-0.00	-0.00	0.00	0.00	-0.00	1.00

Topics

- Statement of the issues
- Geographicals to Cartesians to Geographicals
- Heuristic explanation of correlation problem
- The Molodensky Model at the geocenter (Helmert)
- The derivation of a 7-parameter shift
- Monte Carlo correlation w.r.t. datum area
- The Molodensky Model at the surface (M-B) solves correlation problem
- **Dilution of Precision (P7DOP)**
- Conclusion
- Reversibility in Molodensky-Badekas (Appendix)

Dilution of Precision

Evaluating Adjustment Geometry



Helmert 7-P Dilution of Precision

From the world of hydrography and GPS

$$HDOP = \sqrt{\sigma_E^2 + \sigma_N^2} \Big/ \sigma_0 \qquad PDOP = \sqrt{\sigma_E^2 + \sigma_N^2 + \sigma_U^2} \Big/ \sigma_0$$

$$GDOP = \sqrt{\sigma_E^2 + \sigma_N^2 + \sigma_U^2 + c^2 \cdot \sigma_T^2} \Big/ \sigma_0$$

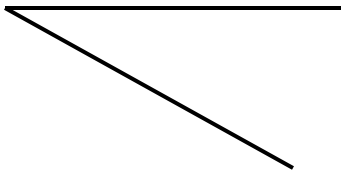
From the world of 7-p Helmert transformations

$$P7DOP = \sqrt{\sigma_{\Delta X}^2 + \sigma_{\Delta Y}^2 + \sigma_{\Delta Z}^2 + a \cdot b \cdot (\sigma_{RX}^2 + \sigma_{RY}^2 + \sigma_{RZ}^2 + \sigma_{\Delta S}^2)} \Big/ \sigma_0$$

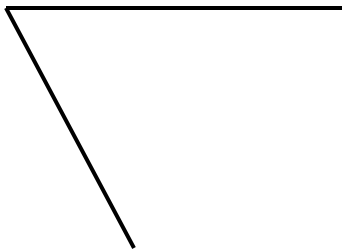
2-D Angle of Cut Analogy

Evaluating Adjustment Geometry

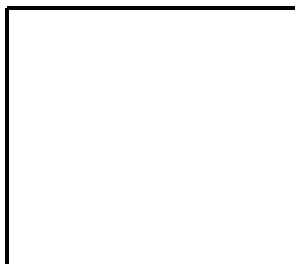
30° HDOP=**2.83**



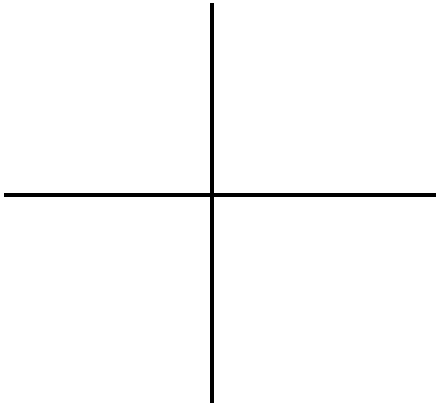
60° HDOP=**1.63**



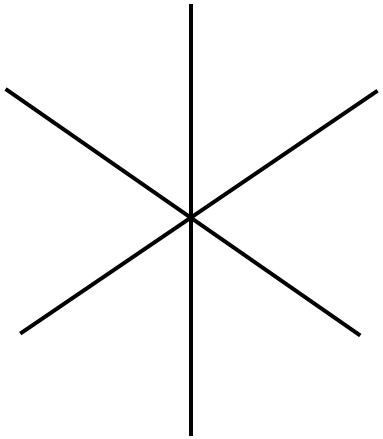
90° HDOP=**1.41**



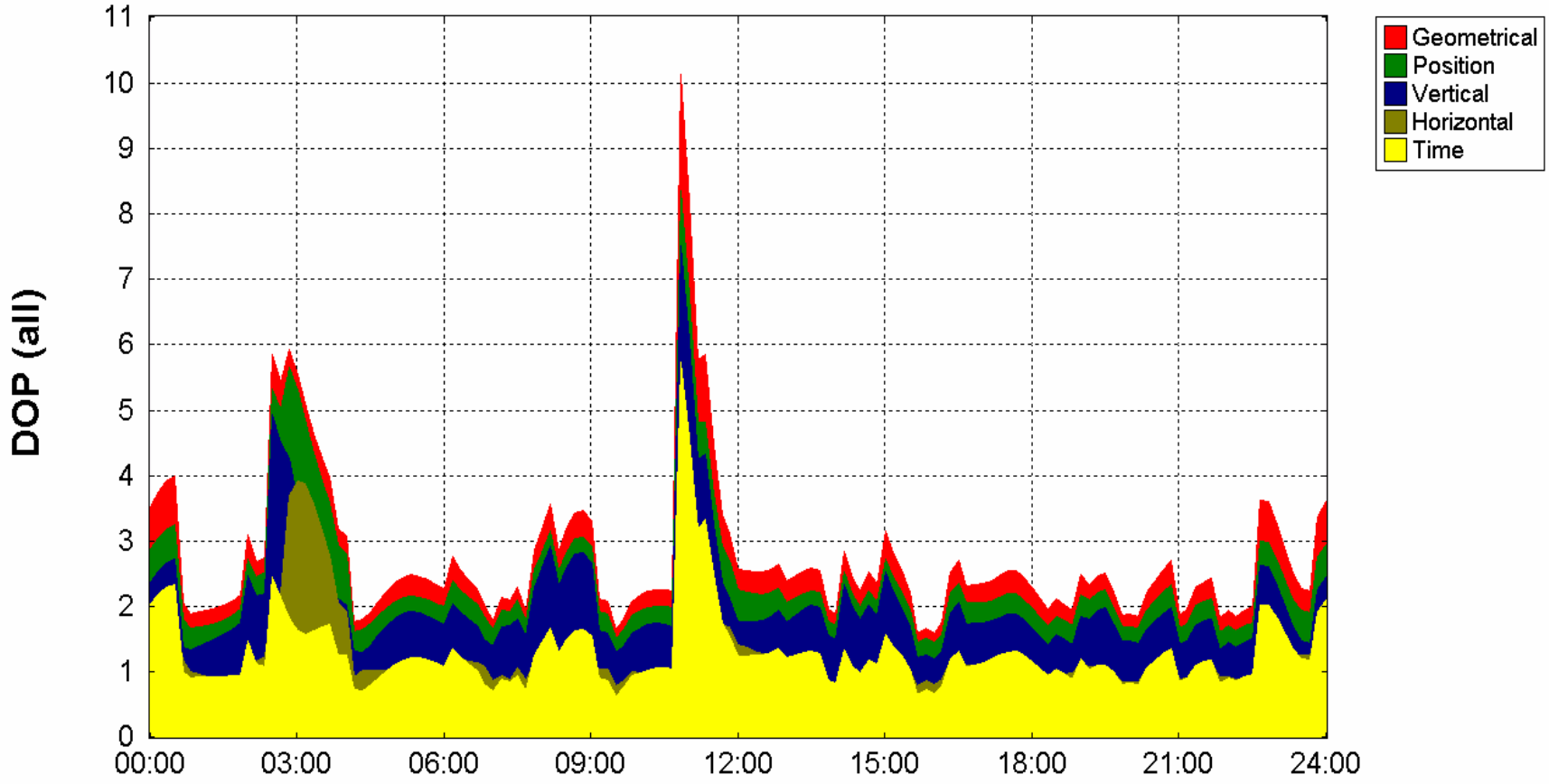
4 at 90° HDOP=**1.00**



6 at 60° HDOP=**0.82**



All GPS DOPs: Houston 2/14/04



7-Parameter Helmert

P7DOP by Area, Angle and Number of Points

P7DOP							
Area	% of World	Deg (+/-)	Number of Points				
			20	40	80	160	320
World	100	180	0.7	0.5	0.3	0.23	0.16
Hemisphere	50	90	1.0	0.7	0.5	0.3	0.2
Russia	3.35	21.1	3.1	2.1	1.5	1.0	0.7
Australia	1.51	14.1	4.5	3.1	2.2	1.5	1.1
India	0.64	9.2	7.0	4.8	3.3	2.3	1.6
Nigeria	0.18	4.9	13.1	9.0	6.2	4.4	3.1
Germany	0.07	3	21	14.6	10.2	7.1	5.0
Ger No Sea	< 0.01	1.2	53	37	25.4	17.8	12.6
Cyprus	0.002	0.5	128	88	61	42.8	30.2

Conclusion

- Due to parametric correlation, Helmert 7-P derivations are global or large-area solutions.
- Small-area 7-P Helmert derivations result in statistically-insignificant parameters, however efficacious in practice.
- Molodensky-Badekas reduces correlations and increases significance in small areas.
- If our industry needs better small-area datum-shift accuracy than 3 parameters provide, the APSG should support Molodensky-Badekas.

Appendices

- Reverse Molodensky-Badekas Equations
- Reverse M-B Numerical Assessment 1
- Reverse M-B Numerical Assessment 2
- Geographicals to Cartesians
- Cartesians to Geographicals
- References

Forward Molodensky-Badekas

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_i = \begin{bmatrix} U \\ V \\ W \end{bmatrix} + \begin{bmatrix} \Delta U \\ \Delta V \\ \Delta W \end{bmatrix} + \begin{bmatrix} 0 & \varpi & -\psi \\ -\varpi & 0 & \varepsilon \\ \psi & -\varepsilon & 0 \end{bmatrix} \begin{bmatrix} U - U' \\ V - V' \\ W - W' \end{bmatrix}_i + \Delta S \begin{bmatrix} U - U' \\ V - V' \\ W - W' \end{bmatrix}_i$$

$$x = u + \Delta u + R \cdot (u - u') + \Delta S \cdot (u - u')$$

Reverse Molodensky-Badekas

$$u_r = x - \Delta u - R \cdot (x - u') - \Delta S \cdot (x - u')$$

$$\boxed{u_r = u + [R^2 \cdot (u' - u) - R \cdot (2 \cdot \Delta S \cdot (u - u') + \Delta u) + \Delta S^2 \cdot (u' - u) - \Delta S \cdot \Delta u]}$$

Round-Trip Error: Numerical Assessment 1

Fictitious "Worst Case"

- **ΔXm** • **+700m**
- **ΔYm** • **-500m**
- **ΔZm** • **+200m**
- **RX"** • **-3"**
- **RY"** • **+5"**
- **RZ"** • **-2"**
- **ΔSppm** • **+3ppm**

Barycenter

```
X   =   3655727.05m
Y   =    373465.14m
Z   =   5194453.82m
Lat =    54.905007d
Lon =     5.833039d
Hgt = -1176.849142m
```

Dist km	X dif m	Y dif m	Z dif m
146.56	0.0020	-0.0054	-0.0090
79.22	0.0020	-0.0054	-0.0091
63.48	0.0021	-0.0054	-0.0091
54.42	0.0021	-0.0054	-0.0091
182.60	0.0022	-0.0053	-0.0091
193.61	0.0022	-0.0053	-0.0091
141.44	0.0022	-0.0053	-0.0091
57.33	0.0021	-0.0054	-0.0091
66.45	0.0021	-0.0054	-0.0091
155.94	0.0021	-0.0054	-0.0092
180.01	0.0020	-0.0054	-0.0091
180.93	0.0020	-0.0054	-0.0091
144.05	0.0022	-0.0053	-0.0091
102.29	0.0022	-0.0053	-0.0091
15.08	0.0021	-0.0054	-0.0091
75.51	0.0021	-0.0054	-0.0091
46.36	0.0021	-0.0054	-0.0091
87.70	0.0021	-0.0054	-0.0091
126.00	0.0020	-0.0054	-0.0091

Round-Trip Error: Numerical Assessment 2

Fictitious "Worst Case"

- ΔXm • +700m
- ΔYm • -500m
- ΔZm • +200m
- RX" • -3"
- RY" • +5"
- RZ" • -2"
- ΔSppm • +3ppm

Barycenter

```
X   =     6229725.66m
Y   =     -384479.16m
Z   =      -57315.84m
Lat =      -0.529766d
Lon =      -3.531637d
Hgt = -136544.205467m
```

Dist km	X dif m	Y dif m	Z dif m
412.33	0.0020	-0.0053	-0.0088
1544.86	0.0028	-0.0051	-0.0093
616.05	0.0023	-0.0054	-0.0094
1532.70	0.0018	-0.0050	-0.0078
1073.02	0.0025	-0.0052	-0.0094
750.19	0.0017	-0.0055	-0.0089
827.55	0.0018	-0.0053	-0.0084
848.43	0.0023	-0.0051	-0.0087
469.44	0.0020	-0.0053	-0.0087
1630.66	0.0022	-0.0059	-0.0104
1100.34	0.0021	-0.0050	-0.0082
1808.66	0.0015	-0.0052	-0.0079
2446.82	0.0013	-0.0057	-0.0088
1125.73	0.0016	-0.0057	-0.0091
154.16	0.0020	-0.0054	-0.0091
1811.03	0.0029	-0.0053	-0.0100
1780.19	0.0029	-0.0053	-0.0102
539.49	0.0022	-0.0054	-0.0094
1683.97	0.0019	-0.0059	-0.0102

Geographicals to Cartesians

Given ellipsoid1 semi-major axis (a) and flattening (f), and latitude (ϕ), longitude (λ), and height (h)

$$b = a - a \cdot f \qquad e^2 = (a^2 - b^2)/a^2 \qquad \nu = \frac{a}{(1 - e^2 \sin^2 \phi)^{1/2}}$$

$$X = (\nu + h)\cos\phi\cos\lambda$$

$$Y = (\nu + h)\cos\phi\sin\lambda$$

$$Z = (\nu(1 - e^2) + h)\sin\phi$$

Cartesians to Geographicals

Given ellipsoid2 *a* and *f*, and X, Y and Z Cartesians

$$b = a - a \cdot f \qquad e^2 = (a^2 - b^2)/a^2 \qquad e'^2 = (a^2 - b^2)/b^2$$

$$\nu = \frac{a}{(1 - e^2 \sin^2 \phi)^{1/2}} \qquad p = (X^2 + Y^2)^{1/2} \qquad \theta = \tan^{-1}\left(\frac{Z \cdot a}{p \cdot b}\right)$$

$$\phi = \tan \frac{Z + e'^2 b \sin^3 \theta}{p - e^2 a \cos^3 \theta}$$

$$\lambda = \tan^{-1}\left(\frac{Y}{X}\right)$$

$$h = (p/\cos\phi) - \upsilon$$

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